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GROUND WATER FLOW MECHANISM CONCEPTUALLY

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Abstract: Availability of resources of surface water is limited. It is unavailable at all places. Hence use of these sources is restricted in order to cope with ever increasing need of supply of water for irrigation purposes, industrial work and domestic purposes. Their global distribution results in least vulnerable in comparison to contaminated surface water sources. Ground water performs active contribution in meeting this high demand especially in India which is developing country and its economy is much more dependent of agriculture sector. This sector extends employment to rural people as well as it is major source of countries economy. To strike a balance between resources and development, it is essential that recharging of groundwater water resources and withdrawal there of are performed with proper planning and supervising. To enhance natural replenishment, education of artificial recharge and ground water withdrawal is crucial. Planning and executing systematic management of groundwater sources are required to overcome scarcity of water on one end and suppress environmental issues on other side affecting all around.

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1 Introduction

Literature is replete with information regarding groundwater distribution on the planet. It is a well-known fact that “Water is distributed within Earth, on its surface, and also in atmosphere in liquid, solid and gaseous forms respectively. Of total available water on Earth, 97% is saline water which is stored in oceans and only remaining 3% is in form of fresh water. Out of this 3% of fresh water, ground water constitutes only 30.1%. remaining are in form of icecap and glaciers (68.7%), liquid surface water (0.3%) and in atmosphere and living being (0.9%). Of liquid surface fresh water, 87% is stored in lakes, 11% in swamps, and only 2% flows in to rivers. Thus, groundwater constitutes second largest reserve of fresh water available on Earth (Wikipedia-a free encyclopedia)”. Soil absorbs fresh water coming from rain, melting ice or snow. It gets accumulated in small tiny pores in sediments, sedimentary rocks and fractured pores. Groundwater contributes to 95% of nation’s fresh water resources. It is stored underground for decades, centuries and thousands of years. 0.6% of hydrosphere is represented by rivers, stream, lakes, ponds and wetlands. Groundwater is generally free from contamination and cleaner than surface water. It emerges to surface as spring or accessed by wells. Groundwater contributes 50% of municipal, domestic and agricultural water supply. Subsurface can hold water for day or for millennia as per length of flow path.

Groundwater is importance source of drinking water which is basic equipment of life on earth. It is essential for domestic, industrial and irrigation purpose. Groundwater plays major role to meet the rising demand of water, specially in countries like ours’ which primarily depends on agriculture for subsistence to rural population and it is main source of livelyhood. It is safe and non-vulnerable source as it is free from contamination. Groundwater is valuable source because of its physical and chemical quality, as it contains few suspended solids, very small concentration of bacteria and virus, also contains dissolved mineral salts with minimal concentration. Hence groundwater is pillar of human life and ideal source. Groundwater does important geological works as that of Uranium and Lead-Zinc deposits. Due to maturation of organic matter deep in sedimentary basins, it results in formation of oil, whereas three-dimensional ground water system helps in migration of this oil and get accumulated in petroleum reservoirs. By process of salinization, soil formation and alteration of soil takes place. Groundwater has a major role to play in this geological process. High concentration of soluble salts is available in saline soil so; it is deleterious for growth of many plants. Thus, groundwater has great importance by ecological point of view. The geophysical science is bridge, as hydrology closely connected with biological science, geographical science and geological science. Assessment of surface water-ground water interaction plays major role in understanding some of geophysical phenomenon. On basis of specific characteristics of water sources bodies and their analysing methods hydrology is broadly classifies as follows:

1. Chemical hydrology: Exploring and studying chemical properties of water
2. Hydrogeology: Studies of presence, distribution and flow movement of ground water through soil and rocks referred as aquifer
3. Eco hydrology: Studies of interactivity between living things and hydrological cycle
4. Hydro informatics: Studies of interrelationship between implementation of information technology for studying hydrological phenomenon and management and uses of resources of water
5. Hydrometeorology: Studies of transportation of water and hydrological power between water zones and lower atmosphere where all weather phenomena take place
6. Isotope hydrology: Studies of isotopic properties of water
7. Catchment hydrology: Studies of fluctuations observed in catchment area by analysing interrelationship between water bodies and resources of ground water present in catchment.

To summarize above, maximum number of models with distinct applications varies from small catchments models to large global models. Every model is unique in characteristic properties and mode of applications. These models are widespread and are built on physical phenomenon of underneath hydrological operations and dispensed over space and time. Application of these models can be traced in area as scaled and unscaled catchments, forecasting of flood, appropriate water resource management and analysing water qualitatively, for studying process of sedimentation and erosion, exploring process of circulation of nutrient and pesticide, innovations related to land use and changes in climate. Catchment serves as main unit of landscapes management and is very important factor for decision making. Hill slopes are fundamental components of catchments which plays major role in understanding procedure of catchment hydrological operations by establishing rainfall-runoff model covering whole of catchment. By considering focus of above all aspects it's clear that groundwater is multi-tasker. It deals with socioeconomically problem and ecological problem. On one hand it will solve problem of drinking water and on other hand it will work in direction of sustainable development by protecting resources.

Study of groundwater is cost affected. To carry out constructed wetland study and cleaning up of contaminated water on land surface is very costly. This gave birth to hydrology modelling. In view of this, it is necessary that basic principles of interaction between surface water and groundwater should be understood systematically, which is require for effective management of water resources. It is well established fact that Surface-groundwater models are of great importance due to its implications in estimation of base flow, effective management of groundwater resources, catchment hydrology and process of recharging and dewatering of aquifer and phenomenon of solute transport across coastal aquifer system.

2 Hydrology and Modelling

Hydrology is important subject for human being and surrounding environment. According to the literature water supply (both surface and groundwater), wastewater treatment, irrigation, drainage, flood control, erosion and sediment control, salinity control, pollution reduction, and flora and fauna protection are important fields where application of hydrology witnessed. Term modelling in hydrology indicates inter connection between water, land, climate and soil. Hydrology is majorly employed in practical aspect of architecting hydrological constructions as dam, structuring wetland and their function, mode of water supply, wastewater treatment plants, irrigation, designing flood control system and pollution control. Model is important constituent of methodical theories. Models are developed in such way that they are snapshot of idealized logical structure. It is representation of existing part of nature or part of human created world in form of Physical, Analog or Mathematical model. Scaled down form of real system is termed as Physical model. Resultant of stimulation process which represents natural process is termed as Analog model. Model which consists of chronological group of relations, numerical calculations and logical computations which transforms numerical input into numerical outcome is called as mathematical modelling.

Modelling of specific facts of real world could assist hydrologist, considerably in griping hydrological problem. Organization and synthetization field data can be achieved accurately and efficiently with help of modelling. In fact, real-world problems like design and structure of dam and drainage system, sewage system, flood flow outlets problems can be resolved with help of these models. Earlier these models serve as good decision-making tool for implementation of project. In reality, flow across percolative medium is three dimensional in nature where as many aquifers are geometrically thin in comparison with their own planar dimension in territorial scale as result models are formulated using much more simple approach referred as hydraulic approach. As per this approach models are formulated under assumptions that flow within aquifer is necessarily horizontal and its vertical component is neglected. First step of modelling of ground water flow is to understand physical problem conceptually. Very next step is translation of considered physical problem in mathematical model associated with set of mathematical constraints as initial condition and boundary conditions and governing equation of groundwater flow. Solution obtained explains[16] flow mechanism of water table as response to hydrological stresses namely recharging, discharging, pumping activity, seepages and surface water and ground water interactions. In general, water flow over ground surface is deterministic character, its magnitude and time span of different processes is stochastic. Hence for efficient and accurate mathematical approach deterministic procedure is combined with stochastic procedure. Generally, groundwater movement equations are partial differential equations with numerous infinite solutions. To arrive at unique solution for considered problem, extra information of considered problem is required, distinct aquifer parameters estimate, the geometry of flow domain, rate of leakage, replinishment rate, withdrawal rate, boundary conditions, initial conditions etc. Based on on physical constraint of considered problem.

3 Preliminaries of Groundwater Hydrology

Following are definitions of few important terms used in groundwater hydrology:

(i) **Groundwater**: It lies below ground surface and is filled partially or completely in tiny pores between rock, sand, soil and gravel.

(ii) **Hydrology**: Study of science of water is called hydrology. It is branch of science that deals with occurrence, movement, distribution and circulation of water beneath ground surface, above ground and even in atmosphere.

(iii) **Saturated zone**: Zone in which all pores, rocks and fractures are filled with water, is called phreatic zone or saturated zone.

(iv) **Unsaturated zone**: It is the hydrological zone which is situated immediately beneath the land surface and consists of water and air is called unsaturated zone or vadosa zone. Moisture present in unsaturated zone moves water in downward direction towards water table to recharge groundwater. Thus, unsaturated zone is of great importance to hydrologist.

(v) **Water Table**: As shown in Figure 1, the uppermost layer of saturated zone is popularly known as water table. It may be immediately below or hundreds of feet below land surface. Process of recharge and discharge from aquifer results in fluctuation of water table.

(vi) **Aquifer**: Group of cenote comprises of geological formation such as saturated rocks or sediments which stores and releases sufficient amount of water for domestic, irrigation and industrial purposes. Aquifers can be regarded as storage reservoir as they possess large areal extent. Aquifer performs two important functions: (i) transportation of ground water to discharge area from recharge area, and ii) lay out storage medium for quantitative usage of groundwater. Quantity of water it can withhold is dependent of porosity of medium.

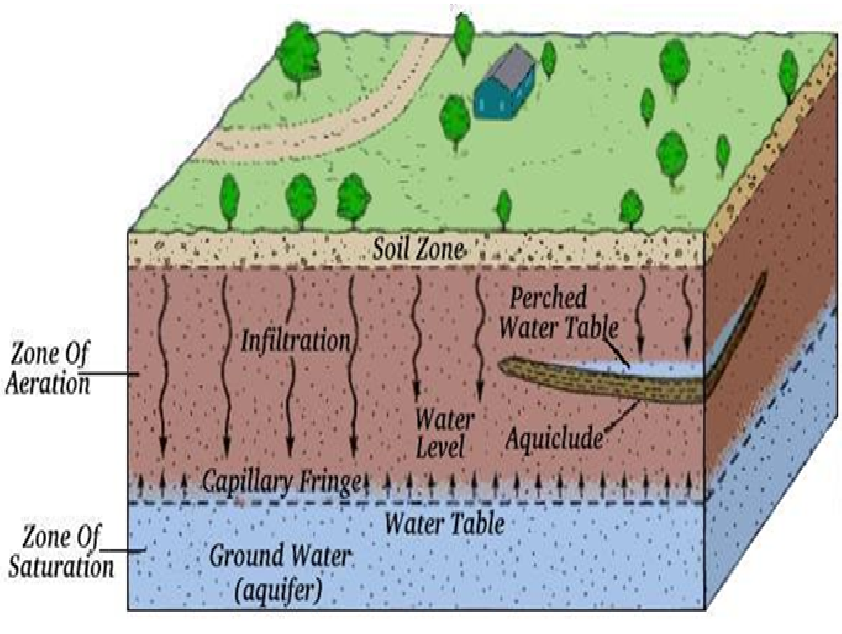

Figure 1. Water table aquifer

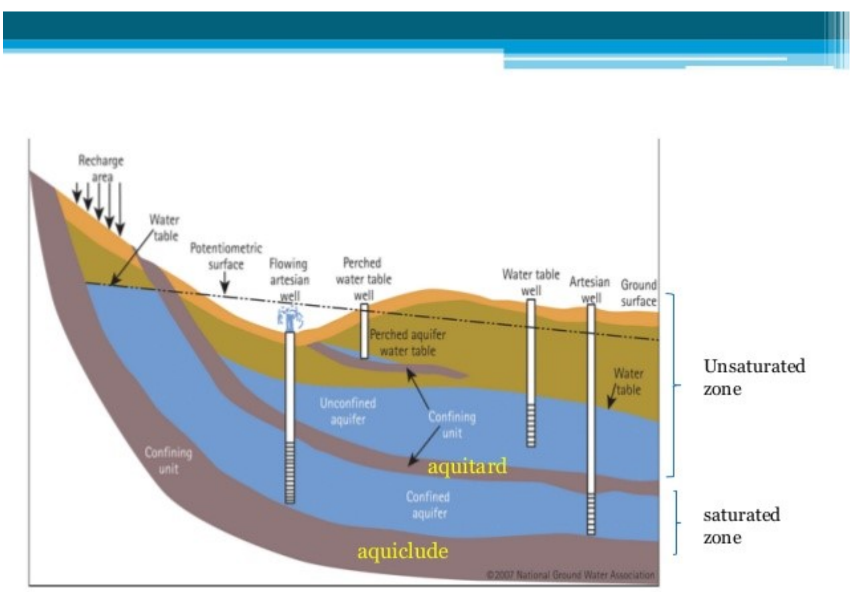

Figure 2. Cross sectional view of aquifer system

For instance, sand, sandstone limestone, crystalline rocks and weathered mantle, highly fractured rocks.

(vii) **Unconfined aquifer**: As shown in figure 2, when aquifer has its upper boundary as water table then it is called unconfined aquifer. It is also called as phreatic aquifer. It is partly filled with water. Additionally, alluded as water-table spring or free groundwater or unconfined groundwater or incidentally called as phreatic water. According to literature Groundwater is not contained under tension underneath moderately impermeable dregs or shakes and is presented to air through openings in overlying soils or in vadose zone. Water table is upper limit of zone of immersion where outright strain and air pressure are equivalent and tension created by water approaches zero. Under unconfined conditions, water table is lot of allowed to rise and fall. During times of dry season, water table might drop as outpouring to springs, stream sand wells diminishing volume of water away. At point when precipitation starts, spring re-energize is for most part fast and water table ascents accordingly. Re-energize of unconfined spring can likewise happen through sidelong groundwater development or through leakage from basic defective bound spring.

(viii) **Confined aquifer**: It is additionally called artesian spring, or restricted groundwater. Development that contains water limited above and beneath by impervious bed. Piezometric strain or head is adequate to make water inside arrangement transcend binding layer, or on account of artesian spring, stream over ground surface. At point when well is introduced through impenetrable layer into restricted spring, water transcends keeping unit. It is sandwich between aquitard above and aquiclude or aquitard below. It is separated from surface by impermeable confining layers or aquitards. It exists between two relatively impermeable layers. It is completely filled with water.

(ix) **Aquitard**: Rock/sediment formation which acts as water barrier by retarding flow of groundwater due to low permebility and low porosity is called aquitard. It permits flow.

(x) **Aquiclude**: Impermeable water barrier which is formidable flow barrier between aquifers is called aqiclude. They have very good storage in comparison with their relatively lower transmission capacity.

(xi) **Aquifuge**: Permeable geological unit neither contains water nor transmits water is called aquifuge.

(xii) **Leaky aquifer**: For aquifer, if there is significant amount of loss or gain of water takes place through semi permeable strata which is underlain, overlain or both, then it is called leaky aquifer.

(xiii) **Ground water recharge**: Natural or intensive infiltration (percolation) of surface water into groundwater system.

4 Parameters of Hydrology

Following are definitions of few important terms used in groundwater hydrology:

1. Porosity: It is defined as the fraction of openings to total volume of soil. In sediments, dependent of size and shape of grains where as in rocks it is dependent of degree of cementation and sorting gradation.
2. Permeability: It is property of material that allows fluids to diffuse through it, is called permeability. It is ease with which movement of water takes place in rock or soil mass.
3. Coefficient of permeability: It is ratio of discharge velocity and hydraulic gradient, and is denoted by K. It is also called as hydraulic conductivity.
4. Specific yield: It is volume added to (or realized from) aquifer per unit horizontal area of aquifer per unit rise of water table.
5. Specific recharge: Amount of water/unit volume stored or expelled from storage due to compressibility of aquifer and pore water/unit change head.
6. Water head: It is height at which pump can raise water up.

7. Specific retention: Ratio of amount of water retained to total volume of saturated aquifer is called Specific retention. It depends on shape and size of particles, pores distribution and structural compactness.
8. Specific storage: It is quality of water per unit volume stored or expelled from storage due to compressibility of mineral skeleton and pore water per unit change in head.
9. Specific storage coefficient: It is amount of water released from storage or moved into storage per unit aquifer storage area per unit change in head. Its unit is m-1. For confined aquifers S varies from 0.00005 to 0.005 and for unconfined aquifers it varies from 0.02 to 0.30
10. Transmissivity (T): It is rate at which water is discharged through unit width of aquifer under unit hydraulic gradient. coefficient of transmissivity is represented as $T = Kb$ (for confined aquifer) and $T = Kh$ (for unconfined aquifer) where b is saturated thickness of aquifer which is equal to depth of confined aquifer and h is equal to mean saturated thickness of unconfined aquifer.
11. Homogeneous medium: If hydraulic properties are same at all location of aquifer, then it is called homogeneous medium.
12. Heterogeneous medium: When hydraulic properties change spatially as well as they are more varying in given direction then it is called as Heterogeneous medium.
13. Isotropic medium: When permeability of point under consideration is independent of direction then such porous medium is called Isotropic medium.
14. Anisotropic medium: When permeability of point under consideration is dependent of direction then such porous medium is called anisotropic medium.
15. Adsorption Accumulation of solid particles as well as ions on surface of soil or rocky structure.
16. Advection Mass transfer of matter by flow of groundwater.
17. Conservative Solute Solute particles which do not have any action on soil particles or rocks is referred as conservative solid.
18. Contamination Stage where groundwater degradation by its quality results in water being unfit for regular consumption is called contamination.
19. Drawdown About given point it's depth by which level of water is lowered.
20. Datum Random surface considered for measurement of hydraulic heads.
21. Diffusion Flow of solute from region of higher concentration to region of lower concentration as result of general movement of molecules is referred as diffusion.
22. Dispersion Spreading of solid particles because of heterogeneous property occurring as resultant of sizes and shapes of pores of aquifer.
23. Dupuit-Forchheimer Assumption According to this assumption vertical flow is neglected in comparison with important horizontal flow.
24. Effective Porosity Section of porosity which is accessible for flow of fluid.
25. Head Conceptually term Head can be explained as relation between potential energy of incompressible fluid and height of static column.
26. Elevation Head It is potential energy of flow of fluid because of fluids elevation above datum of reference. It is calculated as difference between heights of any arbitrary point in flow field and reference datum. Reference datum is generally considered as mean sea level.
27. Equipotential Lines These Lines are lines across which uniform electric potential exists. These are components of Flow net.
28. Force Potential It is expressed as multiplication of acceleration due to gravity and total head. It stands for totality of energy/unit mass.

29. Flow Net It is network formed by intersection of equipotential lines and streamlines at 90 degrees.

30. Head loss Reduction in total head as result of dissipation of energy. Measured in units of depth or height

31. Hydraulic Gradient It is expressed as derivative of hydraulic head with respect to displacement in particular direction. With reference to open channel it is water surface slope.

32. Kinetic(velocity) head It represents K E of fluid. Kinetic head is kinetic energy per unit weight of fluid.

33. Karst In geology it is formation on ground surface made up of limestone or calcium carbonate dissolution in water resulting into underground drainage, sinkholes and caves.

34. Leakage Seeping of water across semipermeable layer namely aquitard

35. Leaky Aquifer geological formation which exhibits seepage of water through overlying semipermeable layer.

36. Longitudinal Dispersion Coefficient: Coefficient of dispersion in direction of flow.

37. Monitoring Well Non-pumping wells utilized for measuring levels of water as well for procuring water samples to perform chemical analysis.

38. Piezometric Head Sum total of elevation and pressure heads.

39. Pressure Head It is flux energy of fluid column whose weight being equivalent to fluid pressure. It is expressed as ratio of pressure intensity of fluid and specific weight of fluid.

40. Reynolds Number Non-dimensional quantity which explains importance of inertial force in context with viscous force of flow system. small Reynolds number indicates laminar type of flow whereas large Reynolds indicates turbulent flow.

41. Streamline line which is tangential to velocity vector of flow at any instant.

42. Total Head Addition of elevation (potential), pressure (potential, and velocity heads. Generally, for flow across percolative medium velocity head being negligible as it is insignificant while computing total head.

43. Velocity Head Kinetic energy of flow per unit weight of fluid; has dimension of length.

5 Problem formulation

5.1 Groundwater flow equations

Groundwater flow equations for a given aquifer system [3] is developed by combining the Darcy's law with mass balance equation. "Henry Darcy gave in famous empirical formula known as Darcy's law which is milestone in ground water hydrology [9, 11]. In 1904, Boussinesq derived nonlinear equation by combining Darcy's law with principle of conservation of mass equation which serves governing equation of groundwater flow in most of research pertaining to surface seepage flow [4, 5]. Darcy's law states that rate flow Q i.e., volume of water per unit time is directly proportional to cross sectional area of porous medium and piezometric head difference between two points $h_1 \breve{} h_2$ and inversely proportional to length of process media L, i.e.

$$Q = k\frac{A(h_1 - h_2)}{L}, \tag{5.1}$$

where K is said to be the hydraulic conductivity of porous media indicating ease of fluid move".

5.2 Flow of groundwater in confined aquifers

If porous medium is inhomogeneous and anisotropic, general ground flow of subsurface seepage in saturated porous medium [15] is given by

$$\frac{\partial}{\partial x}(K_x\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y\frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(K_z\frac{\partial h}{\partial z}) = S_s\frac{\partial h}{\partial t}. \tag{5.2}$$

where x,y, z which are orthogonal coordinate direction. K_x,K_y,K_z are components of hydraulic conductivity in x, y and z directions respectively, S_s is specific storage and h is piezometric head. If medium is homogeneous, then above equation reduces to

$$K_x\frac{\partial^2 h}{\partial x^2} + K_y\frac{\partial^2 h}{\partial y^2} + K_z\frac{\partial^2 h}{\partial z^2} = S_s\frac{\partial h}{\partial t}, \tag{5.3}$$

For isotropic medium $K_x = K_y = K_z = K$ (say), therefore equation (5.3) reduces to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K}\frac{\partial h}{\partial t}. \tag{5.4}$$

If aquifer is of constant thickness b, then storage coefficient S is given by $S = S_s b$. Introducing transmissibility $T = b_k$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{\partial t}, \tag{5.5}$$

or

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}, \tag{5.6}$$

where T is transmissibility. For horizontal flow, $\frac{\partial h}{\partial z}$. Thus, groundwater flow equation in a confined aquifer which is homogeneous, isotropic and is of a uniform thickness is given by

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0. \tag{5.7}$$

For steady flow in homogeneous, isotropic, confined aquifer

$$\frac{\partial}{\partial x}(K_x\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y\frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(K_z\frac{\partial h}{\partial z}) = 0. \tag{5.8}$$

The steady-state ground water flow equation in heterogeneous and anisotropic confined aquifer system is

$$\frac{\partial}{\partial x}(K_x\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y\frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(K_z\frac{\partial h}{\partial z}) = 0. \tag{5.9}$$

6 Flow of groundwater in unconfined aquifers

The unconfined flow problems can be made simple by applying Dupuit-Forchheimer assumptions, further it will be easy to obtain tractable analytical solution [19]. These assumptions are:

- Aquifer base is a horizontal plane.
- The flow of groundwater towards pumping well is horizontal with no vertical hydraulic gradient.
- The horizontal component of hydraulic gradient is constant with depth and equal to water slope.

Using these assumptions, Boussinesq represented equation which is unconfined flow in its transient form with very small and negligible vertical gradient [boussinesq, 1904]. Following cases can be deduced from equation (5.9):

Case-1: Equation of groundwater flow in inhomogeneous, anisotropic unconfined aquifer

As per Dupuit-Forchheimer assumptions vertical flow is neglected in unconfined aquifer and thus in reality motion of fluid is subdivided into component of flow parallel to X-axis and component of flow parallel to Y-axis. Therefore, resulting equation will be expressed as

$$\frac{\partial}{\partial x}(K(x,y)h\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K(x,y)h\frac{\partial h}{\partial y}) = S_y\frac{\partial h}{\partial t}. \tag{6.1}$$

where h represents hydraulic head, $K(x,y)$ represents mean hydraulic conductivity and S_y is specific yield of water table aquifer.

Case-2: Steady state groundwater flow equation in homogeneous, anisotropic, unconfined aquifer

As aquifer storage is unchanged with respect to time in unconfined aquifer, this leads to $\frac{\partial h}{\partial t} = 0$. Therefore, equation (6.1) can be simplified as

$$\frac{\partial}{\partial x}(K(x,y)h\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K(x,y)h\frac{\partial h}{\partial y}) = 0. \tag{6.2}$$

Case-3: Equation in homogeneous, isotropic, unconfined aquifer

If porous media is homogeneous and isotropic, steady-state groundwater flow equation reduces to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0. \tag{6.3}$$

Neglecting vertical components of flow velocity, transient groundwater flow equation in homogeneous and isotropic unconfined aquifer is expressed as

$$\frac{\partial}{\partial x}(h\frac{\partial}{\partial x}) + \frac{\partial}{\partial y}(h\frac{\partial}{\partial y}) = \frac{S_{y_a}}{K}\frac{\partial h}{\partial t}. \tag{6.4}$$

where S_{y_a} is apparent specific yield.

Equation (6.4) is a second order parabolic nonlinear partial differential equation, called as nonlinear Boussinesq equation. If the model domain consists of source/sink, then equation (6.4) can be expressed as

$$\frac{\partial}{\partial x}(h\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(h\frac{\partial h}{\partial y}) + Q(x,y) = \frac{S_y a}{K}\frac{\partial h}{\partial t}, \tag{6.5}$$

where $Q(x,y,z)$ is the recharge or accretion rate.

GWF equation in unconfined slope aquifer

Dupuit-Forchheimer assumptions are basic fundamentals for GWF analysis in unconfined slope aquifer. It is assumed that:

(i) For small inclination of water table, streamlines are approximately parallel to inclined bed.

(ii) Hydraulic head does not vary with depth for small change in inclination of line of seepage.

(iii) Hydraulic gradient causing flow is equal to slope of free surface.

Consider downward slanting unconfined aquifer [8] with small bed slope $\tan\phi$. Let $H_w(x,t)$ represents the height of free surface above the horizon x-axis, $h_w(x,t)$ represents height of free surface above impervious bedding, measured in direction vertically upwards then

$$H_w(x,t) = h_w(x,t) - x\tan\phi. \tag{6.6}$$

Darcy's law over saturated thickness h_w yields expression for flow per unit width as

$$q = -Kh_w \frac{\partial H_w}{\partial x}, \tag{6.7}$$

The flow rate q expressed per unit width of aquifer along x-direction is expressed as

$$q = -Kh_w \frac{\partial}{\partial x}(h_w - xtan\phi). \tag{6.8}$$

Using equation (6.8) in equation (6.9):

$$\frac{\partial q}{\partial x} - R = -S\frac{\partial H_w}{\partial t}, \tag{6.9}$$

or as

$$\frac{\partial}{\partial x}\left(-Kh_w \frac{\partial}{\partial x}(h_w - xtan\phi)\right) - R = -S\frac{\partial h_w}{\partial t}, \tag{6.10}$$

or as

$$K\frac{\partial}{\partial x}\left(h_w\left(\frac{\partial h_w}{\partial x} - \tan\phi\right)\right)h_w + R = \frac{S}{K}\frac{\partial h_w}{\partial t}. \tag{6.11}$$

where R is vertical accretion rate, S is the storage coefficient. On further simplification equation will be expressed as

$$\frac{\partial}{\partial x}\left(h_w \frac{\partial h_w}{\partial x}\right) - \tan\phi\frac{\partial h_w}{\partial x} + \frac{R}{K} = \frac{S}{K}\frac{\partial h_w}{\partial t}. \tag{6.12}$$

Equation (6.12) is a nonlinear parabolic equation of second order. It represents one-dimensional groundwater flow equation in unconfined sloping aquifer which is referred to as the Boussinesq equation. Let downward inclinations of bed respectively along positive directions of X-axis and Y-axis be $\tan\phi_x$ and $\tan\phi_y$. Thus in the Cartesian coordinate system, the two-dimensional groundwater flow equation is represented as

$$\frac{\partial}{\partial x}\left(h_w \frac{\partial h_w}{\partial x}\right) - \tan\phi_x\frac{\partial h_w}{\partial x} + \frac{\partial}{\partial y}\left(h_w \frac{\partial h_w}{\partial y}\right) - \tan\phi_y\frac{\partial h_w}{\partial y} + \frac{R}{K} = \frac{S}{K}\frac{\partial h_w}{\partial t}. \tag{6.13}$$

Mathematical model comprises of governing groundwater flow equation [6], initial condition, and boundary conditions. The mathematical statement for existing constraints of the boundary of the aquifer domain region is termed as boundary condition. These conditions help in gaining more knowledge about the impact of inward flow and outward flow of the domain. These varieties of groundwater flow equations will be subjected to different boundary conditions, namely:

- **Dirichlet's boundary**: It represents a specific hydraulic head value at the boundary. Mathematically, it is represented as:

$$h(x, y, t = 0) = h(x, y, t \to \infty) = h_0, \tag{6.14}$$

  where h_0 is the initial hydraulic head.
- **Neumann's boundary conditions**: It provides information about the specified flux across the boundary. It is represented as $Q_{(x,y,z,t)} = Q_{(constant,t)}$. The no-flow boundary condition is a special case of the Neumann boundary condition where the flux is set to be zero.
- **Cauchy's boundary condition**: For a given value of hydraulic head, flux across the boundary is computed. It is called a mixed boundary condition [13], which is the relation between boundary flux and boundary head.

For transient equations, hydraulic head varies with the function of time, so it is necessary to specify an initial condition. This condition supplies estimates of hydraulic head at all points inside the aquifer domain under consideration before the start of the simulation, that is, at the initial time equals zero.

7 Solution Methodology

Under certain geophysical conditions, surface and groundwater interact continuously. Aquifers in hill slope regions are underlain by sloping impervious bed [14]. Over an inclined aquifer, groundwater transport mechanism is represented as Boussinesq equation in its nonlinear form. The governing equation of groundwater flow is a partial differential equation (PDE) which can be solved numerically or analytically. Groundwater flow problem is converted from PDE to ODE by employing Laplace transform, Fourier transform, Henkel and Mellin transforms [17]. These integral transforms [16] must be selected depending on the coordinate system. The governing groundwater flow equation is a nonlinear PDE of the second order whose exact solution is non-tractable. The occurrence of nonlinearity is due to the coefficient h associated with the partial derivative in the first term on the left-hand side. Thus, to obtain an analytical solution, it is necessary to linearize the flow equation by different methods namely:

(i) Baumann's method (baumann1952)

(ii) Werner's method (werner1951)

(iii) Brutseart's method (brutseart1994)

Consider the water flow equation

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) - \tan\theta\frac{\partial h}{\partial x} = \frac{S}{K\cos^2\theta}\frac{\partial h}{\partial t}. \tag{7.1}$$

In Baumann's method, the h term in the first bracket of the above equation is replaced by the mean saturated depth $h_{\text{avg}} = \frac{h_0+h_f}{2}$ where h_0 and h_f are the initial and final water depth.

In Werner's method, linearization is achieved by linearizing h^2 in the Boussinesq equation:

$$K\cos^2\beta\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) - K\sin\beta\cos\beta\frac{\partial h}{\partial x} + N = S\frac{\partial h}{\partial t}, \tag{7.2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial h^2}{\partial x}\right) - \tan\beta\left(\frac{1}{h}\frac{\partial h^2}{\partial x}\right) + \frac{N}{K\cos^2\beta} = \frac{S}{K\cos^2\beta}\left(\frac{1}{h}\frac{\partial h^2}{\partial t}\right), \tag{7.3}$$

Replacing h^2 by z:

$$\left(\frac{\partial^2 z}{\partial x^2}\right) - \tan\beta\frac{1}{\sqrt{z}}\frac{\partial z}{\partial x} + \frac{N}{K\cos^2\beta} = \frac{S}{K\cos^2\beta}\frac{1}{\sqrt{z}}\frac{\partial z}{\partial t}. \tag{7.4}$$

Now term $\sqrt{z}$ is replaced by average saturated depth h_{avg} so above equation changes as:

$$\left(\frac{\partial^2 z}{\partial x^2}\right) - \tan\beta\frac{1}{h_{\text{avg}}}\frac{\partial z}{\partial x} + \frac{N}{K\cos^2\beta} = \frac{S}{K\cos^2\beta}\frac{1}{h_{\text{avg}}}\frac{\partial z}{\partial t}. \tag{7.5}$$

Next, in Brutseart's method, the term associated with the partial derivative $\frac{\partial h}{\partial x}$ in the equation is replaced by pD where D represents the thickness of the saturated aquifer at the initial stage and p is a constant of linearization. This is an alternate linearization scheme to just above linearization method.

$$\left(\frac{\partial^2 z}{\partial x^2}\right) - \tan\beta\frac{1}{pD}\frac{\partial z}{\partial x} + \frac{N}{pDK\cos^2\beta} = \frac{S}{K\cos^2\beta}\frac{1}{pD}\frac{\partial z}{\partial t}. \tag{7.6}$$

Here, an extra parameter p which can be evaluated in the calibration process or it can be obtained by comparing with available exact solution.

7.1 Analytical solution of linearized Boussinesq equation

- **Laplace transform**: is widely used for solving diffusion-type DE which contain the first-order differential of time.
- **Fourier transform**: It is a powerful tool used in solving BVPs with infinite boundaries. Mixed Fourier transform can be employed in solving linearized Boussinesq with boundary condition [18] and initial condition.

- **Hankel transform**: It is a method which is a powerful tool used for solving groundwater flow equation in polar or cylindrical conditions.
- **Mellin transform**: The groundwater flow equation with free boundary conditions can be solved by using Mellin transform.
- **Green Function**: PDE and ODE, as the case could be solved pertaining to integral terms accompanied by the fundamental function called the green function.

7.2 Semi-analytical solution of boussinesq equation

- **The Adomian decomposition method**: This method is a more reliable and effective method which enables an analytical solution in terms of infinite convergent power series. It provides a more generalized and realistic solution of non-linear PDE and ODE without resolving it into linearized or perturbation method [2]. So, this method is more valuable and is of great importance for hydrologists, physicists, and engineers. It consists of decomposing of unknown terms in series of known terms [1] (Adominan 1994).
- **Semi-analytical methods**: The homotopy perturbation method will be of great aid in obtaining semi-analytical solutions.

7.3 Numerical solution of nonlinear Boussinesq equation

The conservation laws are represented by Partial Differential Equation (PDE). This numerical method transforms Partial Differential Equation (PDE) into discrete algebraic equations over finite elements or cells [7]. It has a flexible approach towards the shape of the boundary. If shapes of boundaries are irregular, then an analytical solution is not tractable. Numerical techniques result in a more realistic solution. It is one of convenient, flexible and powerful tools in solving problems of well hydraulics. Depending on the numerical technique(s) employed in solving the mathematical model, there exist several types of numerical methods used for solving the nonlinear Boussinesq equation. They are as follows:

- **Mac-Comarck Finite difference method**: The hyperbolic PDE is solved by this method. In this method 3-D & time-based derivation are replaced by forward variance to obtain predictor value and corrector value is obtained by replacing space derivation by backward difference. This method is conditionally more stable and convergent.
- **Finite volume method**: The conservation laws are represented by Partial Differential Equation (PDE). This numerical method transforms Partial Differential Equation (PDE) into discrete algebraic equations over finite elements or cells. It has a flexible approach towards the shape of the boundary.
- **Finite-element method**: It is one of the renowned techniques to derive the solution of the partial differential equation delineating groundwater movement. This technique is popular as well as of great potential for solving controlling PDEs. According to literature FEM [12] was already applied in the early 1950s to problems of solid mechanics, however, in later years of the 1960s, the same method is employed to solve groundwater transport problems. In the early period of the 1970s when hydrologists viewed groundwater flow from mathematical modeling perspectives, then they discovered that the application of finite difference method to solve advection-dispersion problem resulted in deviation numerically to some degrees. So Finite Element Method involving integral technique in spite of differential approach is preferable.

The term element in the name of method itself formed by the decomposition of the main region into subdomains. Nodes are defined as corners of these elements. This method is applied for simulating steady-state condition flow systems as well as unsteady-state condition aquifer performance. In this method, every step results in N algebraic set of equations in N variables whereas these N variables represent values of the distribution of hydraulic head for group of nodal points across aquifer. Nature of nodal grids is major factors which differentiate this method from other numerical methods. The main benefit is for simulation in anisotropic medium. Combination of

PDE and calculus of variation is basis for developing finite element equation corresponding to each node.

8 Conclusion

Develop new mathematical model which will account for surface groundwater interaction subjected to various complex [19] hydrological condition [10]. The groundwater movement in unconfined aquifer is monitored by nonlinear partial differential equation referred as Boussinesq equation. The principal goal is to obtain solution of nonlinear equation by applying Adomian Decomposition method which will yield more generalized and more realistic solution without resolving it into linearized form or perturbation method. To develop few real-life subsurface seepage model by overcoming limitations of existing models. These models can be used for calibration of numerical models and guidelines for future experimental studies.

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