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A MODEL-BASED APPROACH TO MULTIDIMENSIONAL INTERPOLATION PROBLEMS

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Abstract: In this article, the problem of forming a sufficient source of information for the optimal management of a discretely defined multidimensional technological object was considered as a problem of multidimensional interpolation. An algorithm for developing a mathematical model of multidimensional interpolation using an empirical method is proposed. The idea of achieving interpolation accuracy based on the requirements for the adequacy of regression models was put forward. Based on the results obtained in the course of the research, it was possible to calculate the interpolation values of the received signal with very high accuracy according to several parameters of the multi-signal forecast models belonging to the same system. Methodologically, the main focus of the research is on building an interpolation function for an object with many parameters and a sufficiently large amount of information and its success.

Keywords and phrases: Multidimensional technological object, interpolation problem, interpolation formula, empirical model, multidimensional interpolation algorithm, discrete information

MSC 2010 Classifications: Primary 93A30; Secondary 00A71.

1 Introduction

It is very important to ensure the continuity, accuracy, and completeness of information in the management of multidimensional technological objects. The increase in the number of object measurements in space and time coordinates creates the problem of optimal control of the information flow in certain intervals of the time abscissa for a discretely defined object. Logically, solving this problem is brought to the formulation of the problem of multidimensional interpolation. Usually, the interpolation problem is considered for limited, sufficiently small experimental results of a pair of variables, i.e., two input and output parameters, and in the end, the performance is highly evaluated [1]. It is noteworthy that the error in the process of one-dimensional "simple" interpolation increases with the increase in the number of input values [2,3]. In fact, it is very difficult to ensure the continuity of information even for one-dimensional "simple" discretely defined objects. This situation becomes more difficult for multidimensional object research. This confirmation can be found in the sources [4]. In general, there is no clear way to solve the problem of multidimensional interpolation as a basic classification of the problem. Methodological uncertainty also requires checking the degree of logical consistency in the statement of the problem. In other words, this question is reasonable, i.e. "is the logic of existence of the problem of multidimensional interpolation, essentially a proven aspect of its existence?". To justify this, it can be considered sufficient to first pay attention to the parametric properties of the object, i.e., the changes in its observable parameters. After all, the reaction of the object is formed under the influence of several equally important input parameters. It is known that parametric control of a multidimensional object is carried out by controlling all important parameters [5,6,7]. In general, as much as information supply is important in the optimal management of multidimensional technological objects, the issue of processing this information is equally important. In the absence of a dominant methodology for solving the interpolation problem for a discretely defined multi-parameter object, we consider it an effective approach to develop a mathematical description of multi-parameter interpolation using highly approximated multi-variable mathematical models.

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2 Materials and methodology

Researchers Nick Lindemulder and Emil Lorist considered interpolation in n-dimensional Banach spaces. The authors explain their research as follows: “We develop a discrete framework for the interpolation of Banach spaces, which contains the well-known real and complex interpolation methods, but also more recent methods like the Rademacher, γ- and q-interpolation methods. Our framework is based on a sequential structure imposed on a Banach space, which allows us to deduce properties of interpolation methods from properties of sequential structures. Our framework has a formulation modelled after both the real and the complex interpolation methods. This enables us to extend various results, previously known only for either the real or the complex interpolation method, to all interpolation methods that fit into our framework. As applications, we prove an interpolation result for analytic operator families and an interpolation result for intersections” [8]. Researcher Yulong Zhang compares different radial basis function interpolation algorithms in radiation field reconstruction and path planning, and investigates multidimensional interpolation problems [9]. In their research, Spiros Zafeiris and George Papadakis develop a redundant interpolation algorithm for multiphase flows using 3D multiblock polyhedral meshes. This approach is effective only when there is a possibility of geometric tracking of multidimensional objects [10]. Researcher Ao Liu and a number of other scientists have studied multidimensional interpolation to map polychlorinated biphenyls in soil [11]. University of Strathclyde researcher P.D. Kaklis P.D. and in the work of a number of scientists, shape-preserving interpolation models on surfaces using splines of variable degree have been developed, and criteria for shape-preserving interpolation on smooth surfaces based on two geodesic curvatures have been proposed [12]. Researchers Thomas Lamby and Samuel Nicolay apply Boyd functions as a multidimensional interpolation model. This is how they interpret the result of this practice: “We generalize here the notion of interpolation space of given exponent by replacing this exponent with Boyd functions. In particular, this approach leads to the usual interpolation method with a function parameter. We present some results in this general setting. Some are well-known, others not so well” [13]. Also, R. Pasupathi and Radu Miculescular develop a way to extend the classical Barnsley framework by allowing for Edelstein and other types of reductions in their research [14]. In the course of our research, the empirical modeling method was used in the construction of mathematical models reflecting the interpolation formula in a multidimensional manner. Also, the comparative method of analysis, similar approach methods were used. The method of least squares was used to parametrize mathematical models, the method of MAPE was used to determine the parameters of distribution and to check the general monadity of the model. Visualization and tabular methods were used to present the results.

3 Main part. Statement of the scientific problem.

We enter these designations for a multidimensional object:: Y_j – output parameters, $j = 1, \ldots, k$; X_i – input parameters, $i = 1, \ldots, m$; let the number of experiments for the system parameters be equal to n. For the relevant s values of i it is required to estimate the state of the object in the x_s^* values corresponding to the arbitrary interval $[x_{s0},\ x_{s1}]$ from the set X_i.

In this case, the following connection is appropriate:

$$Y_{jp} = F_j\left(X_p\right) + \omega, X_p \in \{X_i,\ i = 1, \ldots, m\}, p \le m \tag{3.1}$$

Here, ω is an approximate functional deviation, and in a special case, it is equal to 0 for a one-dimensional object. However, for multidimensional objects, the probability of the residual being equal to 0 is very low, and we assume for now that the output varies with several input parameters. So, practically speaking, in order for the expression (3.1) to be the interpolation formula of the above requirement, the absolute-relative error (MAPE) of the object parameters for the fixed nodal values is required to meet the maximum positive condition [15,16]. It is known that the optimal criterion indicator of MAPE is not higher than 4% [17, 18]. Also, the MAPE indicator is calculated according to the following formula:

$$MAPE = \frac{1}{n}\left|\frac{Y - \tilde{Y}}{Y}\right| * 100\% \tag{3.2}$$

In that case, the construction of (3.1) is carried out using the method of least squares, which is widely used in determining the empirical relationship. Only here, it can be considered that the levels of linear connection [19,20,21] for the structural variables of the model are not important. Because, in our case, the quality of the model, its significance, and the condition of being free from random output quantities lose their significance against the background of satisfying the MAPE<4 condition of (3.2). That is, it is a sufficient condition that (3.1) represents a highly approximable curve in our case. That is, it is a sufficient condition that (3.1) represents a highly approximable curve in our case. However, it is required that the mathematical model in form (3.1) should consist of all important parameters. However, the signs selected as highly important sometimes become insignificant in the empirical model according to the level of significance [22,23]. This situation can be said to be important only if one takes into account the subsequent dynamics of the object over time. In our case, the issue of interpolation is being considered, and it is worth noting that it is related to the calculation of uncertain values in the field belonging to a defined discrete set of objects.

Now we will consider the stage of development of the interpolation model of a multidimensional object. This is done based on the following algorithm;

Step 1. The output parameter of the object (Y) and the input parameters affecting it (X) are separated;

Step 2. (3.1) the bond structure is selected. Here it is appropriate to be based on the principle of simplicity;

Step 3. (3.1) the model is individually parameterized (model coefficients are determined);

Step 4. The MAPE indicator is determined from relation (3.2) for the constructed model;

Step 5. The MAPE indicator is checked according to the 4% criterion;

Step 6. From step 2, the process is continued according to the new scenario (here, this operation is mandatory when the failure of scenario 1 is observed, but it may not be performed on the contrary);

Step 7. If step 6 is performed, the calculated MAPE indicators are compared across scenarios;

Step 8. A suitable model based on the smallest MAPE is adopted and this expression is formalized as an interpolation formula;

Step 9. A mathematical model is used.

This algorithm provides an opportunity to solve the problem of interpolation of multidimensional objects using the method of mathematical modeling.

4 Analysis and results

We consider the parameters of the process of oil product separation as a multidimensional object. We have the following designations: Y - oil consumption at the exit of the separator, cubic meters; X_1 - oil consumption at the separator inlet, X_2 - oil pressure at the separator inlet, atm; X_3 – separator consumption. The distribution of these indicators is presented in Figure 1. Here, one can observe a relatively close distribution of the input parameter of the 1st order to the resulting factor. In general, what is the significance of the parameters being affected by the distribution curves. Let's stop at that. Usually, the creation of a new trend of a result indicator with the help of influencing elements is characterized by the fact that these parameters have a normal distribution in general [24,25]. This is very important for the following:

1) According to the trend. The fact that the structural variables of the model are mutually correct, or create an inverse growth or decrease trend, may be an indication of the high correlation density between them;

2) Equal distribution of parameters over time means that the trend is influenced by laws;

3) The fact that the development of the parameters not only in time or in some monitored control tacts has a similar result means that they are formed based on a certain rule. This aspect is important in the analysis of non-dynamic information.

In our case, it is worth noting that these parameters do not create a separate qualitative trend (Fig. 1). The essence of this is that it is impossible to empirically express the control object dynamically according to the results of the selected experiment.

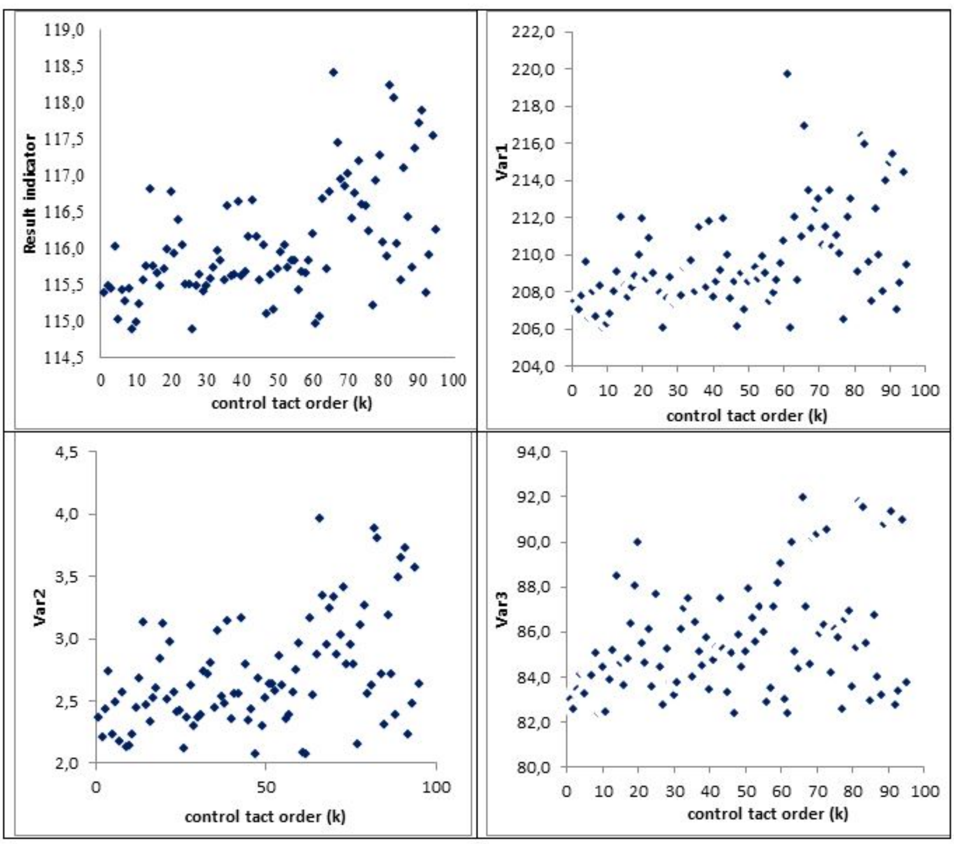

Figure 1. The degree of distribution of discretely determined multidimensional object parameters.

The following empirical models were developed in the input-output system based on the discrete values of these parameters (Fig. 2).

A simple linear regression model was considered as the 1st model. In the model, the variable of order 1 is becoming insignificant. However, in our case, this structural variable remains in the model, because, as noted above, this model is an interpolation function, and a certain effect of this variable on the result is embodied in the accuracy of information. Only here the MAPE indicator is checked. If its value does not exceed the specified rate, it will be possible to evaluate our decision as correct. According to the calculation results, this indicator for model 1 is not greater than 0,13%.

As model 2, an empirical model based on heteroscedasticity was considered. All variables are significant in the model. According to the calculation results, this indicator for model 2 is not greater than 0,13%. In both cases, the MAPE is not greater than the criterion value we presented above, but rather smaller by a large margin. For clarity, $MAPE = 0,129\%$ in model 1 and $MAPE = 0,1276\%$ in model 2. It can be seen that in both cases this indicator is approximately equal, with a slight advantage in model 2.

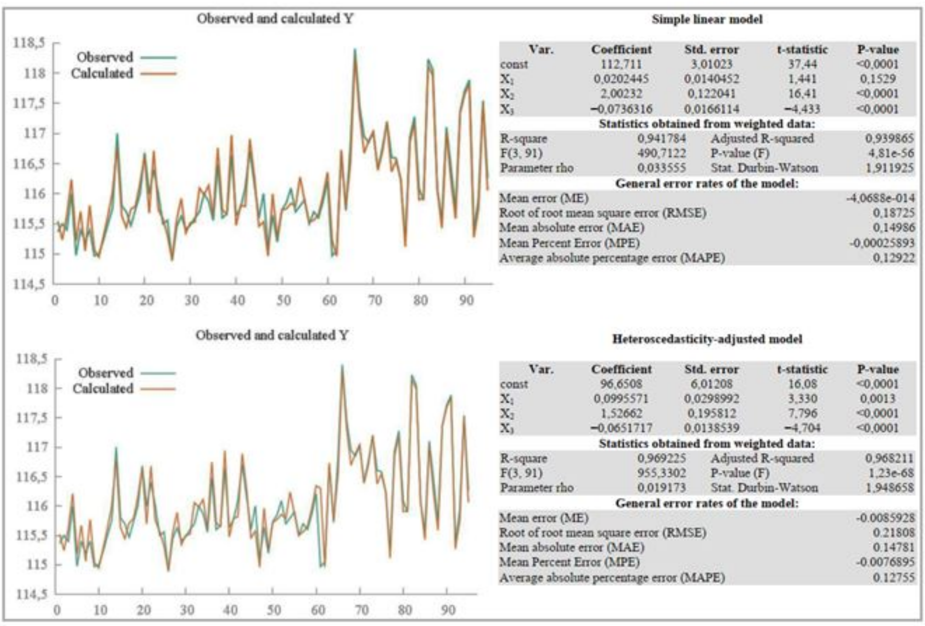

Figure 2. Multifactor Empirical Models, Adequacy Parameters and Interpolation Line for Multivariate Interpolation.

We check the results of interpolation with the developed models. For this, we use the experimental results that were not used in the development of the model (Table 1). In this case, the calculations were made according to the 2nd model.

It can be seen that the calculated values differ little from the real values. In particular, the highest degree of deviation corresponds to the calculation result of the 6th order number. The residual is not greater than 0,002. The average value of residuals is 0,000277, and the sum of squared residuals is not greater than 0,0000002.

In the next case, we check the preservation of the monotonicity condition around the point where the highest error is observed. Because the values selected for the test have intervals of monotonic increase and decrease. For this, we select the range of values in the 5th and 6th order number. That is, as follows:

Selection on the 1st exogenous: $X_1 = \{209,2;;\ 210,2;;\ 211,2\}$Selection on the 2nd exogenous: $X_2 = \{2,7;;\ 2,8;;\ 2,9\}$

Selection on the 3rd exogenous: $X_3 = \{86;;\ 87,\ 88\}$

The calculation results according to model 1 are as follows:

The result of the 1st calculation on the 1st egg: $Y_1 = 116,02$

The result of the 2nd calculation on the 1st egg: $Y_2 = 116,17$

The result of the 3rd calculation on the 1st egg: $Y_3 = 116,31$

The calculation results according to model 2 are as follows:
The result of the 1st calculation on the 2nd egg: $Y_1 = 116,00$
The result of the 2nd calculation on the 2nd egg: $Y_2 = 116,18$
The result of the 3rd calculation on the 2nd egg: $Y_3 = 116,37$
Calculations for both models are approximately the same and monotonically increasing. Indeed, the selected intervals had a growth curve. This exactly satisfies another property of the interpolation line [26,27].

Table 1. Interpolation results calculated using a multivariable regression model

Tests	Var1 (X_1)	Var2 (X_2)	Var3 (X_3)	Real value	Calculated value	The rest
1	206,030	2,124	82,364	114,970	114,885	0,000737223
2	206,240	2,138	84,429	115,000	114,996	0,000036283
3	206,800	2,229	82,428	115,230	115,228	0,000015047
4	208,000	2,444	83,864	115,500	115,573	0,000632237
5	209,100	2,679	85,189	115,760	115,764	0,000033193
6	212,000	3,134	88,474	117,000	116,813	0,001599348
7	208,300	2,467	84,628	115,800	115,754	0,000393199
8	207,700	2,337	83,644	115,700	115,660	0,000342529
9	208,220	2,528	84,814	115,470	115,479	0,000075181
10	208,800	2,605	86,384	115,740	115,721	0,000166436
11	210,000	2,842	88,034	116,000	115,990	0,000089169
12	211,920	3,119	89,959	116,680	116,768	0,000753927
13	208,700	2,517	85,499	116,000	115,935	0,000561453
14	210,890	2,976	84,605	116,400	116,398	0,000017920
15	209,000	2,563	86,112	116,000	116,043	0,000368531
16	207,800	2,409	83,585	115,500	115,513	0,000116193
17	207,900	2,419	87,664	115,560	115,508	0,000451351
18	206,060	2,122	84,431	114,890	114,887	0,000024440
19	207,600	2,368	82,744	115,450	115,482	0,000278788
20	208,768	2,625	85,231	115,630	115,644	0,000118201
21	207,300	2,301	83,754	115,410	115,419	0,000076359
22	207,600	2,362	83,180	115,480	115,495	0,000132704
23	207,800	2,385	83,725	115,600	115,590	0,000082587

5 Conclusion

The following conclusions were reached based on the results of the research on the topic of modern approaches to solving interpolation problems for multidimensional discrete objects: methods and models. Including,

1. Optimum management of a multidimensional technological object can be carried out parametrically in real time by forming a sufficiently complete, continuous, reliable information source. This process can be viewed as a problem of interpolation, and its solutions can form a large-scale information base highlighted;

2. There is no absolute leading methodological approach to the analytical development of a multidimensional interpolation model in the processing of multidimensionality defined discrete data. In our case, the increasing number of parameters and observations is the biggest challenge for any approach;

3. When developing a multi-dimensional interpolation model using multi-symbol regression equations, it is necessary not to take into account the dynamic nature of the object, and the control process should not go out of the specified range of parameters;

4. A methodological criterion for developing a multivariate interpolation model using multivariable regression equations is that the model has a high degree of approximation. Ignoring the importance of coefficients here is based on the fact that the dynamic activity of the object is limited to certain values. That is, modeling is based on the interpolation principle;

5. The main problem in developing a multivariate interpolation model using multivariate regression equations is that sometimes the model does not meet the requirement of high-order approximation. As a result, regression analysis scenarios increase.

6. In practice, the multivariate interpolation function can be replaced by multi-signal empirical forecast models at the level of approximation that does not exceed 4% error.

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References

[1] M.B. Lee et al., Some issues on interpolation matrices of locally scaled radial basis functions, *Applied Mathematics and Computation*, **217:10**, 5011–5014 (2011).

[2] V. Dharmadasa et al., A new interpolation method to resolve under-sampling of UAV-lidar snow depth observations in coniferous forests, *Cold Regions Science and Technology*, **220**, 104134 (2024).

[3] X. Chen et al., Improving the accuracy of wind speed spatial interpolation: A pre-processing algorithm for wind speed dynamic time warping interpolation, *Energy*, **295**, 130876 (2024).

[4] H. Hachiya et al., Position-dependent partial convolutions for supervised spatial interpolation, *Machine Learning with Applications*, **14**, 100514 (2023).

[5] R. Cavoretto R. et al., Numerical cubature on scattered data by adaptive interpolation. *Journal of Computational and Applied Mathematics*, **444**, 115793 (2024).

[6] D. Li et al. Direct cubic B-spline interpolation: A fuzzy interpolating method for weightless, robust and accurate DVC computation,*Optics and Lasers in Engineering*, **172**, 107886 (2024).

[7] M. Sveda et al., When spatial interpolation matters: Seeking an appropriate data transformation from the mobile network for population estimates, *Computers, Environment and Urban Systems*, **110**, 102106 (2024).

[8] M. Lindemulder et al., A discrete framework for the interpolation of Banach spaces, *Advances in Mathematicss*, **440**, 109506 (2024).

[9] Y. Zhang et al., A comparative study of different radial basis function interpolation algorithms in the reconstruction and path planning of γ radiation fields, *Nuclear Engineering and Technology Available*, **56**, (2024).

[10] S. Zafeiris et al., An overset interpolation algorithm for multi-phase flows using 3D multiblock polyhedral meshes, *Computers & Mathematics with Applications*, **161**, 155–173 (2024).

[11] A. Liu et al., Screening and optimization of interpolation methods for mapping soil-borne polychlorinated biphenyls, *Science of The Total Environment*, bf 913, 169498 (2024).

[12] P.D. Kaklis et al., Shape-preserving interpolation on surfaces via variable-degree splines, *Computer Aided Geometric Design*, **109**, 102276 (2024).

[13] T. Lamby, S. Nicolay, Interpolation with a function parameter from the category point of view, *Journal of Functional Analysis*, **286:3**, 110249 (2024).

[14] R. Pasupathi et al., A very general framework for fractal interpolation functions, *Journal of Mathematical Analysis and Applications*, **534:2**, 128093 (2024).

[15] F.D. Jörayev, M.A. Ochilov, Algorithms for multi-factory polynomial modeling of technological processes, *Chemical Technology, Control and Management*, **2023:1**, 2181–1105 (2023).

[16] A.N. Rakhimov, F.D. Jörayev, F. D. (2022). A systematic approach to the methodology of agricultural development and the strategy of econometric modeling, *Res Militaris* **12:4**, 2164–2174 (2022).

[17] A.K. Juraev et al., Nonlinear control object identification problems: Methods and approaches, *E3S Web of Conferences*, **392**, 02043 (2023).

[18] F.D. Juraev et al., Algorithms for improving the process of modeling complex systems based on big data: On the example of regional agricultural production, *E3S Web of Conferences*, **392**, 01050 (2023).

[19] F.D. Juraev et al., Analysis of functions of belonging and assessment of the state of the control object, *World Economics and Finance Bulletin*, **23**, 85–90 (2023).

[20] F. Juraev, Perspective problems of development of agricultural production and their econometric modeling, (4), 377-385, *Economics and Education*, **4**, 377–385 (2021).

[21] I. Khalismatov et al., Correlation analysis of geological factors with the coefficient of gas transfer of organizations, *E3S Web of Conferences*, **497**, 01018 (2024).

[22] F.D. Juraev, A model approach to long-term forecasting of electricity supply, *Journal of Integrated Education and Research*, **2:10**, 41–45 (2023).

[23] F.D. Juraev et al., Problems of Management of Technological Systems Under Uncertainty: Models and Algorithms, *Global Scientific Review*, **19**, 39–48 (2023).

[24] F. Jörayev, Agrocluster system optimization methods: uncertainty minimization using algorithm and model, *Economics and Education*, **24:6**, 306–314 (2023).

[25] F.D. Jörayev, et al., Algorithms for improving models of optimal control for multi-parametric technological processes based on artificial intelligence, *E3S Web of Conferences*, **460**, 04013 (2023).

[26] A.R. Mallaev et al., Improvement of control models of closed systems using neural networks, *Innovative technologies*, **51:3**, 12–26 (2023).

[27] M.A. Ochilov et al., Analysis of important factors in checking the optimality of an indeterminate adjuster in a closed system, *Journal of Critical Review*, **7:15**, 1679–1684 (2020).

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